# Service Scaling on Hyper-Networks

By

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**Abstract**

Service scaling is concerned with service productivity, and hyper-networks with the design of service scaling. This new model uniquely explains Internet-based economic activities, such as e-commerce/e-business and social networking, which are quintessential new genres of service for Knowledge Economy. These activities possess unprecedented promises for scaling: up (reaching the population), down (personalization), and transformational (new business designs). The concept of ***hyper-networks*** has been proposed recently by one of the authors to help explain the analytical nature of such scaling. It establishes the principle of designing ***multiple simultaneous layers of digital connections*** (networking) of persons and organizations on the same basic networks (e.g., the Internet), and interrelating them through "value worm holes" to inflate value propositions (business spaces) and enable ***massive, simultaneous value cocreations*** across the life cycles of persons and organizations. This paper further analyzes the mathematical properties of hyper-networks in the context of a person's multiple roles in his/her life cycle, where each role sparks a particular network. Agent-based simulation experiments confirm that multi-layered networking (e.g., simultaneous multiple social networks) decreases exponentially the degrees of separation, and thereby increases the possibility for value proposition and cocreation in the community.

**Keywords:** service science, hyper-networks, service productivity, simulation, modeling

## I. Hyper-Networks and Service Scaling

Service science faces a number of challenges including the well-known problem of service productivity. A recent study by Hsu and Spohrer (2009) suggests that digital connections scaling is a basic model for creation of the new service of Knowledge Economy (e.g., Internet-based economic activities), which, through the scaling and transformation, improves service productivity and growth. Hsu (2009) further develops the concept and recognizes the existence of hyper-networks in the new service. Hyper-networks help define the analytic nature of service scaling and thereby help its design. We substantiate the above-mentioned new model with mathematical analysis and simulation in this paper.

A prime case of service scaling on hyper-networks happens to be social networking. Enterprises such as Google, YouTube, and Second Life in this area have taught businesses great new lessons about business. One core lesson is the ease with which they ***scaled up*** to the whole user population (or the whole business space), ***down*** to each user's personal needs, and ***transformational*** to create new business designs (from P2P per se to also integrate with B2C, B2B, Portal, ICP, and so on). These social networking phenomena enrich traditional economic sciences, and are subjects of intensive studies in the field of networks science, which (Watts 2003) helps define. However, social networking is but one, albeit pivotal, genre of Internet-based economic activities. Can e-commerce/e-business and other genres of new service - i.e., digitally connected services as Hsu and Spohrer (2009) define it, in the Knowledge Economy (Boisot 2002, Solow 2000, North 2005) duplicate the same scaling? Can all these activities connect? How can we design the connections? The field needs a new networks scientific understanding of the ***community*** of all the Internet-based activities as a whole, to help answer these questions. The concept of hyper-networks (Hsu 2009) brings the newness into the fore.

In fact, the Internet community is indeed the best example of ***hyper-networks***. It consists of a ***basic network*** of user nodes (persons and organizations), characterized by the physical Internet itself. On top of this basic network (or the digital connections infrastructure), the user persons and organizations have created, either by design or by demand, a myriad of layered networks to perform economic, social and other functions pertaining to the tasks in their respective life cycles. It is arguable that these economic and social networks have opened up many new types of value propositions and led to many new business designs and new genres of service for knowledge-based economies (Anderson et.al. 2006, McKenna 2006, Normann 2001). The rapid growth since Internet's inception is truly amazing: Is the growth sustainable? Will it accelerate even more? How does it power service innovation? Can we design success on the Internet? One needs to understand the analytic nature of the Internet community as a whole, i.e., as a hyper-network, to be able to answer these and other similar questions.

Although only empirical data can prove what has happened, analytic results of hyper-networks can help one interpret, explain, and extrapolate what happened. The analysis includes how single networks (single layers) interact with each other, and inflate their values from these interactions. Previous results in the field are concerned mainly with single networks: how they grow horizontally (expand a single relation) and, to a lesser extent, vertically (adding more relations as separate layers) - see, e.g., Hanneman and Riddle (2006) and Miller and Page (2007). Study of the interrelation of layers and, indeed, even the formulation of multi-layered communities, such as the Internet, has been few and far in between. The concept of hyper-networks provides a formal model to characterize the analytic structure of networks with multiple layers. The notion of layers also support differing abstraction of networks in particular thought models. In comparison, the usual model of social networks comprises of only one layer. Chapter 3 of Hsu (2009) presents a set of propositions and theorems relating hyper-networks to service scaling. We synthesize them in the following basic proposition that this paper studies:

**Service Scaling Proposition:** Growth of Internet-based economic activities comes from three main sources, each of which spawns new value propositions and cocreation. They are (1) the growth of single networks (digital connections of persons and organizations); (2) the addition of new single networks; and (3) the interrelation of layers of single networks along and across a person's and an organization's life cycle tasks. We further submit that the interrelation of networking layers represents the most powerful source of value proposition inflation. The points of interrelation may be metaphorically referred to as the "***value worm holes***" (in the spirit of the "worm hole" concept of astrophysics). A value worm hole is multi-dimensional and multi-layered.

In the Internet community, we recognize value worm holes in Google and many other sites where social networking and e-commerce integrate (e.g., embedded B2C); and thereby explain their stellar growth by this concept. In general, the study of hyper-networks promises to provide insights into the design for service scaling, such as the identification and creation of the "value worm holes" to help expand value propositions and facilitate value cocreation. Furthermore, the concept of hyper-networks expressly recognizes the fact that each person may become both providers and customers of certain value, just as organizations always are. The reason is simple: each person always possesses certain information and resources valuable to others in some context. The secret is, therefore, how to materialize these contexts (value propositions) and then connect them for value cocreation. We submit that such connections spell innovation of service. Clearly, interrelation of network layers connects these contexts and maximizes them for all to benefit. The result is the concept of hyper-networks, where each node can be a versatile partner in all kinds of value cocreation for all kinds of value propositions when the needs of all nodes are connected along their life cycles.

Section II further elaborates and formulates the conceptual model of hyper-networks, while Section III presents their mathematical models and derives their basic analytic properties based on probabilistic laws. Section IV shows a preliminary study of the behaviors of hyper-networks under certain settings, using agent-based simulation. Agents are employed to implement the behavioral models associated with each mode. The simulation results serve as a baseline measure for interrelation of multiple layers based on roles of the behavioral models. The last section, Section V, remarks on the implications of the study and their extensions in the future, and concludes the paper.

## II. Service Scaling Design: The Conceptual Model of Hyper-Networks

**Define** H(B, N, E, M, R), where

H: a multi-dimensional hyper-network defined by parameters B, N, E, M, and P.

B: the basic network of H, defined by N and E and implemented by digital infrastructure.

N: the set of nodes of the basic network (size n)

E: the set of edges defined by available digital connections among the nodes of the basic network (E is of size n(n-1)/2 in an unrestricted network where any node can gain access to any other nodes).

M: the set of a priori defined roles available to N (i.e., each node has up to m defined roles), where each role defines a particular layer of networking out of N and E.

R: the (n x m) behavior matrix where each element is a model governing how the node performs each role. The simplest form for a behavioral model of an element in the matrix is a (n-1) vector of probabilities defining the chance of each node connecting to every other node for a given defined role.

The basic network is the physical foundation of hyper-networking. It does not have to be a layer of networks in the sense of users' logical connections (as in, e.g., social networking); however, it could if the community happens to be constructed this way, as in a campus network. For our purpose, the above model recognizes it in order to highlight the nature of digital connections for hyper-networks – i.e., enabled by a physical foundation which itself is subject to design. Otherwise, it could be indistinguishable from the other layers and be removed from the model. In fact, our analysis (Section III) and simulation (Section IV) do just this: they assume a pervasive basic network in the community and accordingly treat it as a constant outside of the scope of the study.

Each node is a person or an organization, and each edge is a digital connection. When all roles are isolated, then H is a mere collection of r single networks. However, when interrelated nodes and edges are present, then layers of networks are connected and the interrelating nodes become value worm holes for making value propositions and value cocreation across layers. For example, the social networking site Second Life is a "value worm hole" connecting persons whose lives may otherwise never get crossed paths. We review the basic promises of the above conceptual model below, to establish that it supports service scaling design.

A good measure for the power of interrelating network layers is the increase of connection – which opens up channels for value cocreation. Connection, in turn, may be measured by "degrees of separation", or the "small world" phenomenon (Watts and Strogatz 1998, Blass 2004) of the networks science. The Service Scaling Proposition of Section I requires that, first, hyper-networks with multiple roles for persons and organizations allow significantly smaller degrees of separation than single networks; and second, connections lead to possibilities of value proposition and cocreation. At present, we take the second premise as it is - a given. Yet, the first promise may be justified by mathematical analysis and simulation – the task we undertake in the next sections. Before we indulge ourselves with the formal studies, we review the conceptual foundations of the above model.

Hyper-networks can evolve along any and every dimensions, by expanding R and P, as well as B. They are multiple (social and economic) networks convoluted (layered, with interrelation) into one (e.g., integration of B2C, B2B, B2E, P2P, etc.). From the perspective of service science, ***each hyper-network is a multi-dimensional user community for massive (possibly concurrent) value cocreation***. Each dimension corresponds to a networking layer, and a layer characterizes a particular type of ***role*** of the basic network governed by a particular behavioral regime for the nodes. In other words, a hyper-network is a basic network that plays a myriad set of simultaneous roles, each of which gives rise to a particular social or economic network. Each role defines a layer and a dimension of the holistic community, or the hyper-network. In a usual network, each node subscribes to just one behavioral model – which defines its sole role, such as dating, supply chain, and games. This characterization lends itself naturally to multi-agent implementation when we study how it works. In this way, a type of agent represents a particular role for a node when it subscribes to the particular behavioral model. A node, then, is operational when it completes with a set of multi-typed agents to perform its various roles.

Hyper-networks are both descriptive and prescriptive. That is, hyper-networks are subject to design: they can be proactively developed, from first digital connections and then promotion of value worm holes to facilitate massive simultaneous value cocreation among persons and organizations. The interrelation of layer networks amounts to multiple-role playing for nodes in the basic network, where each node is associated with an open number of behavioral models and hence types of role, for various value cocreation in various business spaces enabled by the same basic network. This openness affords the nodes a multitude, multiplex, and indeed multiplication of possibilities beyond the usual sense of scalability of networks. The word inflation might actually better describe this expansion. As such, the interrelation of layer networks promises to be a source of innovation and the opening of new spaces, which awaits exploitation. When hyper-networking is amenable to design, so does ***service scaling*** by virtue of designing hyper-networks: service scaling on hyper-networks. It follows that we study the analytic properties of hyper-networks to gain design knowledge for service scaling. We now explain this point further: why hyper-networks may help define new genres of service in Knowledge Economy, and why it may be central to a new service science (as discussed in, e.g., [Cambridge Papers 2008, Hsu 2009]).

To do this, we turn to the foundation of new service, which features hyper-networks. Service has some commonly accepted defining properties, such as perishable, one-of-a-kind, and co-production (Bitner and Brown 2006, Dietrich and Harrison 2006, Lovelock 2007, Lusch and Vargo 2006, Spohrer et.al. 2008, Spohrer and Maglio 2008, Tien and Berg 200)]. We submit that the quintessential definition is actually the cocreation of value between customer and provider, which is what the Cambridge Papers (2008) embraces. In this sense, service scaling is concerned with promoting value cocreation and the enhancement of service quality and productivity through new and better value propositions [Hsu and Spohrer 2009, Hsu 2009]. This recognition leads to the Digital Connections Scaling (DCS) model for service science (Hsu 2007, Hsu and Spohrer 2009, Hsu 2009).

The DCS model theorizes that digitization makes service resources (people, organization, information, and technology) everywhere in the world connectible, and digital connections can (be designed to) scale up, down, and transformational, to spawn value cocreation for persons, organizations and society. The notions of customer and provider are in fact roles that persons and organizations play in the cocreation. These basic roles may interchange and evolve along the life cycles of the persons and organizations concerned, for the tasks of the value propositions involved at each stage of the life cycles. The scaling of value cocreation is, therefore, a function of the scaling of digital connections among persons, organizations, and society. Such digital connections are comparable to networking except that the networks involved can have numerous purposes and each node (person or organization) in the network can have numerous roles to play towards these purposes, simultaneously. In other words, the communities of value cocreation are really hyper-networks. Their role-based evolution is the layer-based evolution of the underlying hyper-networks.

We should point out that connection is a basic concept in many studies, ranging from organization (Carley 1999) and sociology (Watts 2003) all the way to biology (Kauffman 1993). Although researchers may take it for granted that the connections often employ digital means such as the Internet, many previous results tend to neglect the unique conceptual implications of digital connections. In contrast, the DCS model argues that digital connections are not just one type of connections; rather, they constitute a new world of connection in which service (value cocreation) dominates. In one word, digital connections scale: they can scale up to the population, scale down to personalization, and scale to transform. This again corroborates that a digitally connected world is a multi-dimensional community of many co-existing social and economic networks convoluted recursively into one: the community of hyper-networks. Just as the DCS concept is broad, so does that of hyper-networks. While social engineering borders impossibility for a community of non-digital connections, digitally connected communities are amenable for scaling design pursuing similar goals of value: numerous simultaneous networks can not only evolve into being by themselves, but they also can be facilitated from the basic networks which physically exists, and can operate to support many concurrent roles at any given time.

Since digital connections scaling naturally results in hyper-networks (e.g., the Internet), designing DCS to promote value cocreation and new value propositions is designing hyper-networks. Thus, if digital connections scaling makes any sense at all for the new service genres in the age of knowledge-based economies, which we think it does, then studying hyper-networks is inevitably valuable. As a first study, this paper explores the basic analytic properties of hyper-networks, to provide a basis for studies corroborating the concepts with the empirical evidence on the Internet. The results are presented in the next two sections.

**III. Mathematical Models of Hyper-Networks**

In this section, we derive the mathematical models for hyper-networks with multiple layers and dimensions. We assume that there are $n$ individuals (or, nodes), $p_1, p_2, \ldots, p_n$, in the network. The network is called an $m$-role hyper-network if every member of it plays up to $m$ roles in the network simultaneously. Each role substantiates a logical layer of networking, which may not connect to other layers by itself – i.e., each role is naturally distinct from other roles in the hyper-network.

To establish a logical edge for a role-based network, we employ the simple definition of probability prescribing the chance that a node may connect to another, in the tradition of networks science. That is, each role of an individual is characterized by a popularity factor, $g_{li}$. Individual $p_i$ is acquainted directly with individual $p_j$ under role $l$ with probability $g_{li}g_{lj}$.

Figure 1 gives an example of a two-role hyper-network, which is a combination of a professional network and a family network. Every individual in the hyper-network can be a teacher in his/her professional network while at the same time being a father/mother in his/her family network. The combined networks depict a hyper-network with two dimensions. The figure consists of three graphs: the lower two show traditional views of separate individual social networks, respectively; while the top one shows the hyper-network and its two logical layers.

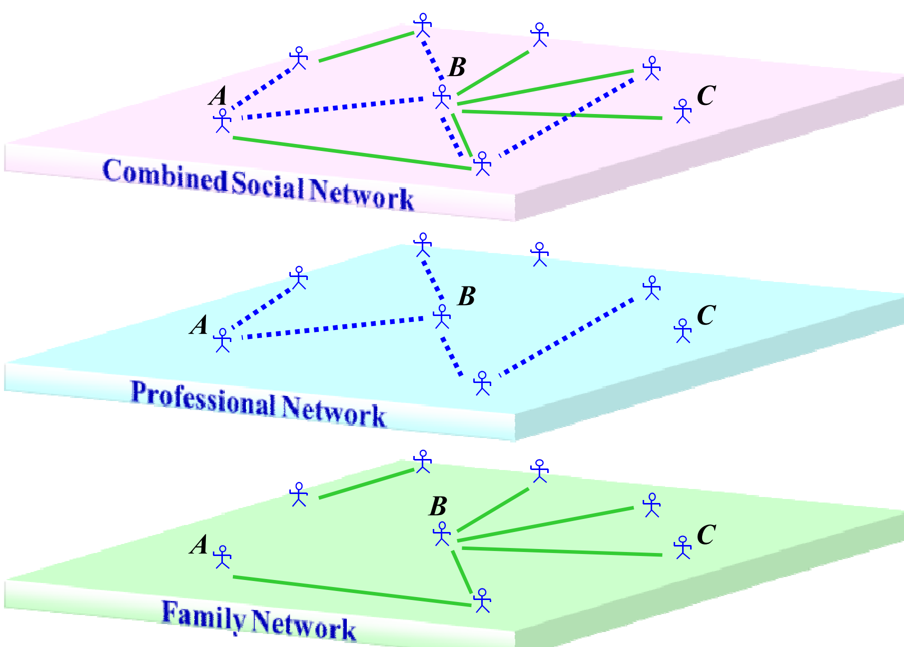


Figure 1. A Two-Role Hyper-Network

As discussed in Section II, the power of multiple roles can be measured by the concept of degrees of separation. Our objective here is to examine how the degrees of separation changes due to the effect of multiple roles. We quantify the degrees of separation between two individuals by using the length of the shortest path between these two individuals. Specifically, if $p_i$ is directly connected to $p_j$ (i.e., known to each other) under anyone of the $m$ roles that they play, the shortest path length between $p_i$ and $p_j$ is one. For instance, in Figure 1 the shortest path between individuals $A$ and $B$ is one under the professional network and two under the family network; the shortest path between them in the whole two-role hyper-network is therefore one. Instead of directly connected to $p_j$, $p_i$ might be connected to $p_j$ through $p_k$. In this case, the shortest path length between $p_i$ and $p_j$ is two. For example, in Figure 1 the shortest path between individuals $A$ and $C$ is three under the family network; however, since $A$ is directly connected to $B$ in the professional network, the shortest path between $A$ and $C$ in the whole two-role network is two (with $B$ as the intermediate node). In general, if $p_i$ is connected to $p_j$ through $x-1$ members, the shortest path length between $p_i$ and $p_j$ is $x$. Let $X_{ij}^{(m)}$ be the shortest path length between individuals $p_i$ and $p_j$ in an $m$-role network, $i, j = 1, \ldots, n$.

**III.A. Analysis of One-Role Networks**

We first consider the case when $m = 1$. The probability that the length of the shortest path between $p_i$ and $p_j$ is one is $\Pr\left[X_{ij}^{(1)} = 1\right] = g_{ki}g_{kj}$. When there is no direct connection between $p_i$ and $p_j$, the shortest path length becomes two if there is at least one intermediate node that connects both $p_i$ and $p_j$ (i.e., $p_i$ connects to $p_j$ through an intermediate node $p_{k_1}$), giving:

$$\Pr\left[X_{ij}^{(1)} = 2\right] = (1 - g_i g_j)\left[1 - \prod_{k_1 \neq i,j}\left(1 - g_i g_j g_{k_1}^2\right)\right]$$

Similarly, the probability that the shortest path length is $x$ (i.e., $p_i$ connects to $p_j$ through $p_{k_1}, p_{k_2}, \ldots, p_{k_{x-1}}$), is:

$$\Pr\left[X_{ij}^{(1)}=x\right]=\prod_{q=0}^{x-2}\prod_{k_1,\ldots,k_q\neq i,j}\left(1-g_ig_j\prod_{l=1}^{q}g_{k_l}^2\right)\left[1-\prod_{k_1,\ldots,k_{x-1}\neq i,j}\left(1-g_ig_j\prod_{l=1}^{x-1}g_{k_l}^2\right)\right],$$

where when $q=0$, the second product reduces to $1-g_ig_j$. Since there are $n$ individuals in the network, the maximum possible intermediate links between $p_i$ and $p_j$ is $n-1$ (i.e., $p_i$ connects to $p_j$ through all the rest $n-2$ nodes in the network).

For completeness, if there is no connection between $p_i$ and $p_j$, we define the shortest path length between them as $n$. The probability of that is:

$$\Pr\left[X_{ij}^{(1)}=n\right]=\prod_{q=0}^{n-2}\prod_{k_1,\ldots,k_q\neq i,j}\left(1-g_ig_j\prod_{l=1}^{q}g_{k_l}^2\right).$$

For ease of exposition, define quantity $Q^{(1)}(x)$ as:

$$Q^{(1)}(x)=\prod_{q=0}^{x-1}\prod_{k_1,\ldots,k_q\neq i,j}\left(1-g_ig_j\prod_{l=1}^{q}g_{k_l}^2\right),\qquad x=0,1,2,\ldots,n-1,$$

where $Q^{(1)}(0)=1$ and $Q^{(1)}(1)=1-g_ig_j$. As a result, we have:

$$\Pr\left[X_{ij}^{(1)}=x\right]=Q^{(1)}(x-1)-Q^{(1)}(x),\qquad x=1,2,\ldots,n-1.$$

In summary, the distribution of $X_{ij}^{(k)}$ is given by,

$$X_{ij}^{(1)}=\begin{cases}x & \text{w.p. } Q^{(1)}(x-1)-Q^{(1)}(x),\qquad x=1,2,\ldots,n-1\\ n & \text{w.p. } Q^{(1)}(n-1)\end{cases}.$$

**III.B. Analysis of $m$-Role Networks**

We now consider an $m$-role hyper-network. The connection between $p_i$ and $p_j$ can appear in anyone of the $m$ roles. The probability that there is no direct connection between them is $\prod_{l_1\in\mathbb{M}}(1-g_{l_1i}g_{l_1j})$. Hence, the probability that there is at least one direct connection (of length one)