Service Value Networks: Humans Hyper-network to Cocreate Value

Wai Kin Victor Chan, Member, IEEE and Cheng Hsu, Member, IEEE

Abstract—Service is about value cocreation between customers and providers. Cocreation builds on human networking: people connecting fluidly with each other as customers, providers, and resources to pursue common values. This paper develops a new analysis of service value networks, building on a previously presented hyper-network model to study how people can scale their value cocreation up to span the entire population (domain), down to meet individual needs, and transformationally to breed new business designs. The new analysis reflects the convergence of social networks and e-commerce, and the evolution of physical products towards incorporating services to users into them (such as the apps and digital resources on the iPod, iPhone, and iPad). The hyper-network model analyzes human networks that overlay multidimensionally, such as the Internet community itself. These properties extend the previous research results on random graphs and semi-regular networks. A simulation study helps verify the hyper-networking analysis.

Index Terms—hyper-networks, network science, social networks, service science.

I. SERVICE VALUE NETWORKS ARE HUMAN-CENTERED

THE defining property of service is value cocreation [1]; i.e., the customer and the provider of a service jointly design and execute the service. In this sense, humans are the center of value cocreation in any service system. They are the end customers, the end workers, and the resources of the value cocreation. Therefore, service systems are all about networking humans in the pursuit of value propositions that are common to them. We adopt the notion of service value networks to refer expressly to this human centric view of value cocreation. The notion is consistent to such definitions as the world is a whole service system for people [1]; service is a dominant logic or perspective that drives enterprises [2]; the provision of any technology, processes, and operations is service [3]; and service is a new mode of production defined by microeconomic production functions that feature digital connections scaling [4].

This paper presents a new analysis for service value networks. It begins with modeling the multiple layers of connection of customers, providers, and resources in service systems as hyper-networks that span all these layers. Then, it extends the previous hyper-network model [5, 6, 7] and the theory of digital connections scaling [8] to obtain unique new insights into how service value networks work analytically. A methodology suggests the directions for scaling service value networks up to the population (as shown in search engine services), down to persons (as shown in personalization services of e-commerce), and transformationally to new business designs (as shown in the rapid evolution of Internet enterprises). The new analysis adds to service science, while the hyper-network model represents a new basic result to network science.

The new analysis is founded on this basic thesis: humans hyper-networking to cocreate values. Here, the key concepts are hyper-networking and collaboration (value cocreation). The Internet has witnessed people collaborating willingly to create knowledge (e.g., blogs and open sources), information resources (e.g., YouTube and Wikipedia), and connections (e.g., Twitter and Facebook), in a manner that tempts one to call it “all for one and one for all”. In industry, knowledge workers similarly network to support each other in professional undertakings within and even across their organizational boundaries. Open technology communities exemplify these powerful networks of professionals. In addition, many organizations also embrace internal social networking as a tool for their employees to boost quality and productivity. Now, all these social networking practices are converging with business, especially e-commerce. Companies enhance their marketing with social networking offerings (comments, blogs, etc.) or participation in social networking sites. Conversely, social networking sites also develop their own revenue-bearing services for people and companies.

Note that all these human networks are intertwined by virtue of a common denominator – the (same) persons who constitute the networks; and all persons are intertwined by the roles they play in their life cycles: being a student, a professional, a spouse, a customer as a parent, a provider as a photographer, etc. Thus, people belong simultaneously to myriads of networks because these networks belong respectively in myriads of roles that persons play. Human networks become service value networks when people cocreate values on them, as a customer, a provider, or a resource, either separately or concurrently. For example,

Manuscript received January 14, 2011. This work was supported in part by the U.S. Department of Commerce under Grant BST23456 (sponsor and financial support acknowledgment goes here).

W. K. V. Chan is with the Industrial and Systems Engineering Department, Rensselaer Polytechnic Institute, 110 8th St., Troy, NY 12180 USA (e-mail: chanw@rpi.edu).

C. Hsu is with the Industrial and Systems Engineering Department, Rensselaer Polytechnic Institute, 110 8th St., Troy, NY 12180 USA (e-mail: hsuc@rpi.edu).
hobbies can become business and business hobbies; and crossing the lines of customers and providers can be as easy as uploading and downloading digital files. The divides are intertwined across personal life cycle roles.

All people consume, thus they by definition are customers. However, many such customers are also providers of the Internet community (e.g., via Wikipedia and YouTube) even when they do not engage in any selling. For formal business, many people use B2C providers (e.g., e-Bay, Amazon.com, and Yahoo!) and/or set up revenue-bearing blogs on social networking sites. Companies network through people, too. They often engage the social networks of executives or involve professionals in open communities, as well as performing such cross-organizational collaboration as B2B practices (e.g., Alibaba.com and GroupOn.com), industrial exchanges (e.g., Covisint.com and Ariba.com), and Internet utilities (e.g., Oasis.com and BBC). Finally, physical products are also increasingly associated with service value networks, as the producers/providers strive to attend to their customers' real reasons for buying the products: using these products to fulfill their life tasks. The evolution of the iPod, iPhone, and iPad well illustrates this point, where manufacturers join forces with service providers (personal apps and information resources) to cocreate values with their customers – i.e., enabling them to network socially and conduct business for making a living as well as for personal tasks.

As such, people and companies create, expand, and combine their service value networks throughout their spheres of existence – i.e., they hyper-network their customers, providers, and resources along multiple dimensions, on demand. The evolution of Yahoo! and Google from providing Web search and email to providing e-commerce and now social networking are examples of this hyper-networking. They scale up to the population since they have to accumulate the population resources (Web pages). They also scale down to individuals to provide personalized values as competitive weapons. In so doing, the population resources they accumulated not only enable them to pursue innovative (role-based, task-based) personalization, but also become their fundamental strategic assets for launching new business models and designs. Similar stories also come from Amazon.com, which evolved from being a humble online bookstore to, now, an all-encompassing B2C site that provides Web presence, social networking, and computing to other sites. In a similar way, one-stop travel sites such as Expedia.com and Hotwire.com well illustrate the concept of life cycle tasks. They combine the information, transactions, and social networking involved in the life cycle of traveling for travelers who may be a businessman, a tourist, or assuming any of the many possible roles of life. Finally, the story of the iPad portrays hyper-networking across business spaces and industries (e.g., content provider vs. channel provider, and hardware vs. software).

The rest of the paper justifies the above analysis. Specifically, Section II reviews the previous results in the literature, with Section III presenting a methodology for scaling service value networks. Sections IV and V consolidate the mathematical foundations of the hyper-network analysis. They show that incorporating a new customer base or offering new services to an existing customer base is equivalent to creating a hyper-network, and the average distance for hyper-networks is shorter than those for single-layer random graphs and scale-free networks. Simulation is used to help verify these results. Section VI discusses how the hyper-network model may analyze social networks better, while Section VII analyzes the degree distribution of hyper-networks with simulation. Section VIII concludes the paper.

II. THE HYPER-NETWORK MODEL CONNECTS SERVICE SCIENCE AND NETWORK SCIENCE

Network science emerged mainly from the studies of engineering, biological, and social networks. These traditional networks may be categorized as random, semi-regular, or regular according to how the nodes connect. Random networks feature connection laws based only on probability (especially the uniform distribution), while regular networks are fixed in their connection designs (e.g., telecommunications and infrastructure [9]). Thus, regular networks tend to well describe physical systems, and random networks the logical connections among humans. Semi-regular networks include all possibilities in between random and regular. Service value networks are either random or semi-regular, and comparable to social networks except that they include organizations, too.

Although the baseline random network model assumes equal chances for any connection to take place [10], broader interpretations of the randomness are possible. Semi-regular networks recognize constraints on connections, which reflect the realities of human habitats. Technically, all human networking by the Internet should be semi-regular in nature since (access to) the Internet itself presents a constraint. In any case, the distinction between broadly defined random networks and semi-regular networks may be academic, unless constraints are explicitly defined and included.

From the perspective of human networking, we recognize a few basic connection laws, which determine the topology and such performance measures as degree distribution, distance between nodes, and the centrality of nodes within a network [11]. First is the class of random connection laws and their extensions motivated by different conditions of probability [12]. An interesting extension is the exponential connection laws proposed for social networking [13]. For semi-regular networks, including the random graphs that are subject to constraints or exogenous factors of influence, two genres stand out. The first is the Small World networks [5, 6], whose topology features lumps of largely random connections with a small number of “long connections” linking up these lumps. The literature has yet to provide a detailed analysis for the causes of the topology, but a plausible explanation is the natural similarity of nodes, such as biological bonding, geographical proximity, and/or social classes. The long connections reflect exceptions, or outliers, which may be subject to design and management. This class well represents natural congregations [16, 17].

The second genre, the Scale Free networks [7], features a
degree distribution (numbers of connections at a node vs. numbers of nodes having such numbers of connections) that exhibits a decay power law. Scale Free networks seem to be pervasive in any situation where persons can proactively choose associations. For example, the power law phenomena encompass the usual 80-20 rule or the Pareto distribution [21]. We suspect that this pattern may be more basic to collective human activities than previously realized, and may apply to economics (e.g., income distribution), engineering designs (e.g., pivotal parts), and systems integration (e.g., databases inter-operation [22]), as well. In any case, the commonly recognized causes behind the power law phenomena include personal preferences such as connection by popularity or kinship; which are human choice in nature. Therefore, preference may actually reflect a wide range of scientific principles related to choice. As such, the particular forms of preference may be subject to design and management (e.g., opinion leaders, name recognition, and diminishing costs).

All the network models reviewed above consist of only one (two-dimensional) layer. The hyper-network model [5, 6, 7] extends them to multi-dimensional by connecting multiple layers of inter-related single networks to represent the intertwined nature of human networks. The connection laws of hyper-networks are subject to all the results reviewed above, plus its own unique rules which apply across layers: i.e., multi-dimensional connections by roles – the many roles that humans play in their life cycles. It is worth noting here that the wholeness of a hyper-network is not merely a collection of single networks, thus words such as composite and heterogeneous do not fully and accurately describe the whole network. For human hyper-networks, each layer (single network) is but some role-characterized differential existence of the whole person.

The previous hyper-network results [6, 7] show that multiple layer connections significantly reduce the average distance in a network; while adding (random) nodes and connections to a single network transforms it to a hyper-network. The distance formulae of hyper-networks may help explain some of the anomalies reported in the literature, such as the persistent underestimation of acquaintances (i.e., the theoretical distance is larger than what the data shows).

The paper further consolidates and enhances these results and presents an improved hyper-network model for service value networks analysis. The new model is presented next.

III. USE HYPER-NETWORKING TO SCALE SERVICE VALUE NETWORKS

Conceptually, a hyper-network is a network of networks. It represents a connected community in which members can link to each other via multiple types of connection channels in multiple types of networks overlaid on top of one another. The channels could use different mechanisms (ranging from digital devices to social and biological bindings) that tie people together via information or physical exchange. In addition, nodes and their connections may have different significance and relative importance in different layers, and hence the degree of connections may have different meanings from one layer to another. In other words, a hyper-network is a multi-dimensional graph with a possibility of “flavors”. Below is an enhanced definition of a hyper-network:

Define $H(B, N, E, M, R)$, where

- $B$: the optional base network, consisting of $N$ and $E$ to represent the physical domain of $H$.
- $N$: the set of nodes of the base network (size $n$), consisting of all members of the $H$ community.
- $E$: the set of edges, defining the feasibility of connections among all nodes of $B$ ($E$ is of size $n(n-1)/2$ if any node can connect to any other nodes). $E$ is removed if $B$ is.
- $M$: the set of community-common roles available to $N$ (i.e., each node has up to $m$ possibilities of roles), where each common role defines a particular layer (or color) of networking out of $N$ and $E$. $M$ defines the dimensionality of $H$, and its value may be time-dependent.
- $R$: the $(n \times m)$ behavior matrix where each element is a model governing how the node performs each role. The simplest form for an element of $R$ is a $(n-1)$-vector of probabilities that a node may connect to every other node for a given role. $R$ defines the strength of connections, too. The base network represents the physical or regular foundation of the networked community. It is optional to the hyper-network model if the community has no restrictions on connections – i.e., if $E$ is free and random. In general, the behavioral nature of connections between nodes such as feelings and intensity are functions of the $R$ matrices involved. The above definition provides a three- or higher-dimensional graph, with $m$-layers of edges connecting the same set of nodes over a foundation. That is, an edge between two nodes of a network may have any number of “colors” signifying different roles or genres of channels of connection.

The core properties of hyper-networks that the paper proves later are (1) hyper-networks are pervasive in human networks, and (2) they shorten the distance for information exchange among people while expanding the degree of direct linkages for each member. From the core properties we submit that scaling service value networks may correspond to building a hyper-network for the base community of the value cocreation domain. These expansions are commonplace with social networking sites, as they routinely add new tools, applications, or other provisions for socialization. Clearly, business mergers and B2C partnerships are examples, too.

The above model lends itself to a methodology for hyper-networking: apply the connection laws within and across layers to scale service up, down, and transformationally. This methodology aims at non-service sectors as well as the traditional service sector, since hyper-networking opens up the possibility of “humanizing” products by connecting them to their end customer’s individual needs. For example, the iPod, iPhone, and iPad might continue to evolve into an “iWeb” where the whole Web gets packed into a small square device, using cloud computing to provide information and transactions for users in the way they want. For physical products, humanization could mean mass customization of almost anything for almost anyone, from personalized healthcare and designer drugs, all the way to household based alternative
energy, micro smart grids, and even indoor agriculture. In all of these examples, the mass customization relies on hyper-networking of customers, professionals, and resources in the pursuit of value cocreation. This broad concept describes a new mode of production that features scaling of service value networks, or a “servicization” of the whole economy.

The hyper-network connection laws are amenable to implementation by the Digital Connections Scaling approach [5, 8] – i.e., use digital means to connect people for value cocreation at on-demand scales. Digitization reduces the cycle time and transaction cost of connection, and scaling these connections decreases the marginal cost for value cocreation. The following propositions detail the methodology:

**Proposition 1: Create the Accumulation Effect**

(maximum growth: linear, $O(n)$)

The first level of service value network scaling is concerned with building up the sheer size of N and the resources that accompany the members (nodes). The connection laws of Scale Free networks – i.e., the dynamism of preferences, should guide the accumulation. As discussed in Section I, they include name recognition, opinion leaders, and other competitive advantages. Thus, building such preferences will be the core of scaling strategies. We submit that scaling down, or personalization for serving the members’ individual life cycle needs and tasks, is a general tool for scaling up, too, since it helps establish preferences and leads to accumulation; this, in turn, enables new service business designs – see Section I. The growth rate is proportional to the number of nodes, n.

In addition to building preferences, another important form of the accumulate effects is the sharing and re-use of the accumulated resources, customers, and providers. They reduce the learning curves of service systems design and the marginal costs of value cocreation. Many e-commerce models, e.g., Internet Services Providers, Internet Contents Providers, and Application Services Providers have shown these types of benefits [5, 8]. The heavy equipment industry also competes on basis of fleet (population) information to cocreate value in the operation and/or maintenance of the equipment for their clients. The accumulation effect often becomes barriers to entry against new comers, as well as the competitive advantages for these businesses.

**Proposition 2: Create the Network Effect**

(maximum growth: polynomial, $O(n(n-1)/2)$)

The second level of service value network scaling is concerned with building up the number of (role-based) single layer networks for the hyper-networking community. The connection laws of Small World networks may contribute expressly to this scaling; i.e., cultivating long connections to complement proximity (in all forms) of congregation. Proven ways for growing long connections include providing more chances for exceptions and outliers to happen, which may cut across known boundaries of groupings and domains. Examples include the popular e-marketing feature of providing “recommendations”; B2C sites clustering their products (e.g., by product attributes, customer attributes, and behavior attributes) to penetrate customer divides; and the clustering of B2C sites at e-commerce portals (e.g., industrial exchanges). Larger scale practices are found in, for example, open source communities, open social networking sites, and the consulting industry. The promotion of Software as a Service (SaaS), Service Oriented Architecture (SoA), and IBM’s on-demand business represents hyper-networking in the consulting industry.

Since the core logic here is two-dimensional networking, the growth rate is fundamentally proportional to the number of pairs between members.

The next proposition is unique to hyper-networking: It goes beyond the traditional network effect to call for promoting multiple value cocreation networks along the paths of connecting personal life cycle roles and integrating tasks.

**Proposition 3: Create the Ecosystem Effect**

(maximum growth: factorial, $O(n!)$)

The third and highest level of service value network scaling is concerned with the *life cycle tasks* that people and organizations undertake. The connection laws of the hyper-network model drive this proposition. Information and transaction portals in e-commerce, such as those for one-stop travel arrangement and online banking, represent harbingers of this proposition. The practices of embedded B2C at, e.g., Facebook, Google, and Meet.com also reflect this proposition. Broader hyper-networking of customers, providers, and resources will take place if the service value network uses humans’ life cycle roles as roadmaps to pursue the integration of their tasks. Person-specific roles and tasks can lead to scaling down while common tasks and roles can lead to scaling up. Furthermore, integration may lead to transformation and new service business designs. The Small World and Scale Free connection laws may be generalized to guide the pursuit of cross-layer hyper-networking, such as cultivating long connections between layers and promoting preferences with multiple roles (centrality of the hyper-network).

Conversely, hyper-networking may enhance the effects of proximity and preference on single layers. That is, connections of roles may result in additional possibilities of cascading interactions among all members (customers and providers) in the community. This possibility is revealed by the hyper-network property that multiple layers reduce the average distance. Since there are indefinite possibilities for realizing roles, the ecosystem effect may drastically increase the possibility of developing value propositions among persons.

When people interact, they tend to follow some specific sequence; i.e., an interaction between two persons is directional. When an ordered pair connects to other pairs, they form permuted sequences. This is the chaining of human interactions. Thus, the possible number of all interactions is proportional to the mathematical permutations of all members in the community – i.e., the maximum growth rate is factorial on the number of members. Technically speaking, roles are sequence-sensitive, or even sequence-dependent; and role-based hyper-networking allows for asynchronous interactions. Any formula that imposes a fixed maximum of roles on members, such as $n$ to the power of $m$, is bound to under-
estimate the real richness of a role-based human community.

In sum, a human community is an ecosystem of value cocreation based on roles, and hyper-networking describes the analytic nature of the ecosystem. Any two persons could generate any number of value-propositions in between them, with any one being the customer in some of them and the provider in the others. Such pairings can then coalesce in the community to form single layer networks pursuing congruent value propositions. By allowing such networks to intertwine and support each other, hyper-networks result in order to increase value cocreation and decrease marginal costs.

IV. RANDOM ADDITION OF CONNECTIONS IN HUMAN NETWORKS CREATES HYPER-NETWORKS

This section establishes the core properties of hyper-networks: creating multi-layers will reduce the distance (or degree of separation) between persons in a community, and randomly drawing new connections in a single network will transform it into a hyper-network. These properties singularly support the scaling of service value networks, since the proximity of persons affects the ease of value cocreation between them.

In the following analysis, the distance between any two arbitrary persons is defined as the length of the shortest path between their nodes. The degree of a node is the number of immediate neighbors of that person. The network is an m-role hyper-network if there are m possibilities or channels (e.g., social networking sites) to tie any pair of persons together via information exchange. These channels could also be different relationships that a member can explore in the network (e.g., friendship, kinship, and working relationship). Each layer substantiates a logical means of networking, which may or may not connect to other layers by itself.

In a real network, the degree of a node at a particular layer is determined by factors such as activeness, popularity, friendliness, and so forth. For simplicity, we assume that all these factors can be aggregated and represented by a single parameter valued in [0,1] — the “popularity” factor, which is denoted by \( p_{il} \) for individual \( i \) at layer \( l \). This popularity factor embodies the likelihood that a node makes connections and attracts connections. The larger its value, the higher the chance is for a node to have more links. In addition, this chance of connection should be mutual, that is, the existence of a link between nodes \( i \) and \( j \) is proportional to the product of the two popularity factors. Specifically, individual \( i \) is acquainted directly with individual \( j \) in layer \( l \) with probability \( p_{ij} \), which is equal to the product of the two individuals’ popularity factors, i.e., \( p_{ij} = g_{il}g_{jl} \).

The objective here is to examine the effect of multiple layers of connection on the degree of separation. We quantify the degree of separation between two nodes by using the shortest distance between these two persons. Specifically, if individual \( i \) is directly connected to individual \( j \) (e.g., known to each other) through anyone of the \( m \) layers, the distance between them is one. If \( i \) is not directly connected to \( j \) in any layer but indirectly connected to \( j \) through another individual \( k \) in at least one layer, then the distance between \( i \) and \( j \) is two. In general, if \( i \) is connected to \( j \) through \( x \) members in any one or multiple layers, the distance between them is \( x \). If there is more than one path from \( i \) to \( j \), then the distance between them is the minimum distance among all paths in all layers. The path here can traverse different layers. Let \( X_{ij}^{(m)} \) be the distance between individuals \( i \) and \( j \) in an \( m \)-layer network, \( i, j = 1, \ldots, n \), where \( n \) is the number of nodes in the network.

To begin the analysis, consider a 1-layer network and examine how the distance between two arbitrary individuals \( i \) and \( j \) changes when we randomly draw new connections among people with a given probability, on top of the existing network. We argue that such a random addition of connections is equivalent to creating another layer and hence making it a hyper-network.

As shown in [8], the distance \( X_{ij}^{(1)} \) is given by:

\[
X_{ij}^{(1)} = \begin{cases} x & \text{w.p. } Q^{(1)}(x-1) - Q^{(1)}(x), \quad x = 1, \ldots, n-1, \\ n & \text{w.p. } Q^{(1)}(n-1) \end{cases}
\]

where \( Q^{(1)}(x) = \prod_{k=0}^{x-1} \prod_{i=1, i 
eq j}^{k} (1 - \prod_{l=1}^{k} p_{il}k_{il}k_{il}) \) for \( x = 2, \ldots, n-1, \) \( Q^{(1)}(0) = 1, \) and \( Q^{(1)}(1) = 1 - p_{ij} \).

Now, let us randomly add a new connection between nodes \( i \) and \( j \) with a probability \( p_{2ij} \). It is clear that this new connection is equivalent analytically to creating a link at a new layer for this pair if a connection between them already existed. Even when no previous connection exists, this new link still could be a new layer. When more nodes are drawn into the creation of new connections, and some of them are on top of previously existed ones, this group of newly created connections clearly forms a congruent new layer on top of the existing one. In the simplest case, where only one link is to be drawn, it is easy to obtain the distribution of the distance between \( i \) and \( j \). Denote this new distance by \( X_{ij}^{(2)}' \):

\[
X_{ij}^{(2)}' = \begin{cases} 1 & \text{w.p. } [Q^{(1)}(0) - Q^{(1)}(1)] (1 - p_{2ij}) + p_{2ij}, \\ x & \text{w.p. } [Q^{(1)}(x-1) - Q^{(1)}(x)] (1 - p_{2ij}), \\ n & \text{w.p. } Q^{(1)}(n-1) (1 - p_{2ij}) \end{cases}
\]

Therefore, the expected reduction in the average distance is:

\[
\Delta d = E \left[ X_{ij}^{(1)} \right] - E \left[ X_{ij}^{(2)}' \right] = \left( \sum_{x=0}^{n-1} Q^{(1)}(x) - 1 \right) p_{2ij}.
\]

In general, the expected reduction in the distance when a new link is drawn between every pair of persons (i.e., for every pair of \( k_{1}, k_{2}, k_{1} \neq k_{2} \in \{1, \ldots, n\} \)) with a given probability can be calculated as follows. In addition to the original probability \( p_{1k_{1}k_{2}} \), there is another probability, \( p_{2k_{1}k_{2}} \), such that nodes \( k_{1} \) and \( k_{2} \) are connected. This is equivalent to
a 2-layer hyper-network. To see this, consider three arbitrary persons \( i, k_1, \) and \( j \). Suppose in the original network (Layer 1) \( i \) is connected to \( k_1 \) and both \( i \) and \( k_1 \) are disconnected to \( j \). Suppose also that a random link is successfully drawn between \( k_1 \) and \( j \). This not only changes the distance between \( k_1 \) and \( j \) but also links \( i \) and \( j \) through the intermediate \( k_1 \) who is playing two roles - one in the original network with \( i \) and the other one in the new link with \( j \). This is exactly the definition of a hyper-network in which persons are connected via multiple channels. The formula for obtaining the expected reduction in the average distance is:

\[
\Delta \tilde{d}_{12} = E \left[X_{ij}^{(1)}\right] - E \left[X_{ij}^{(2)}\right] = \sum_{x=0}^{n-1} \left(Q^{(1)}(x) - Q^{(2)}(x)\right).
\]

We now use a simulation to examine the effect of randomly drawing connections among a number of nodes. Both pure random graphs (Poisson network) and scale-free networks (power law) of size from 100 to 10K are simulated. We generated the l-layer original network with homogenous connection probability \( p \) (with \( p \) varying from 0.005 to 0.8 in different experiments), and then randomly connect \( U\% \) of the nodes (with \( U \) varying from 0.01 to 0.8 in different experiments). Each experiment is replicated 50 times. Two measures of average distance are employed: the first one assigns a distance \( n \) to all disconnected pairs and averages all distances, while the second one assigns an infinite distance to all disconnected pairs and takes the reciprocal of the sum of the reciprocals of each distance. Their formulae are shown below:

\[
\tilde{d}_a = 2 \sum_{i,j} n_{i,j} X_{ij}^{(1)} \left\lfloor \frac{n_i n_j}{n(n-1)} \right\rfloor
\]

where \( X_{ij}^{(1)} = n \) if \( (i,j) \) are disconnected, and

\[
\tilde{d}_b = \frac{1}{\sum_{i,j} n_{i,j} X_{ij}^{(1)}}
\]

The reduction in the average distance is computed as the difference between the original distance and the new distance (i.e., the distance with random connections added) in units of the new distance, or specifically, \( \Delta_a = (\tilde{d}_a - \tilde{d}_a)/\tilde{d}_a \) and \( \Delta_b = (\tilde{d}_b - \tilde{d}_b)/\tilde{d}_b \), where \( \tilde{d}_a \) and \( \tilde{d}_b \) are the new distances. Figure 1(a) and 1(b) show the percentage of reduction in the average distance in a random graph. The \( x \)-axis is the percentage of persons selected for adding connections. Each line there represents the amount of reduction with original homogenous connection probability varying from 0.005 to 0.8. Figure 2(a) and 2(b) display similar results for scale-free networks with each line there corresponding to different exponents (from 1.6 to 3.4, where a value between 2.0 to 3.0 being the common value observed in most real-world systems). It is evident from the figures that the reduction is more powerful when the original network has fewer connections. Moreover, the reduction in the second measure of distance exhibited similar linearity for both types of networks.

V. HOW HYPER-NETWORKS HELP: THE ESTIMATION FORMULAE

In this section, we derive formulae for estimating the average distance and average degree in a hyper-network. The baseline for our derivation is random graphs, which usually assume homogenous acquaintance probability – i.e., \( p_{ij} = g_{ij} = p, \forall i,j \). The average degree in random graphs is \((n-1)p\) or approximately \(np\) if \( n \) is large.
For traditional networks, there are many representative results for determining network performance in terms of distance and degree. The results cover random graphs [10, 21, 23], scale-free networks [18, 19, 20], which are shown to be consistent with the basic characteristic of small-world networks [24, 25]. A “good-get-richer” generation mechanism for scale-free networks is proposed as an alternative generation mechanism to the usual generation conditions “rich-get-richer” (i.e., preferential attachment) and growth [20].

Now, assume that we superimpose \( m - 1 \) layers on a random network, making it an \( m \)-layer hyper-network, with the 1st layer being the original random network. We further assume that the acquaintance probabilities in the superimposed \( m - 1 \) layers are identical (i.e., \( p_{ijl} = p_l \equiv q, \forall l \neq 1 \)). If \( q \) is significantly smaller than \( p_l \) (i.e., \( p_l \gg q \)), then each of the \( m - 1 \) layers can be viewed as a virtual social community that links certain groups of people together. Please note that for simplicity of mathematical derivation, we have combined all characteristics of a link into one single parameter \( p_l \) (or \( q \)), although in reality links exhibit various characteristics [9].

Denote the average degrees of a hyper-network and an ordinary Poisson network by, respectively, \( < z_h > \) and \( < z > \). When \( n \) is large, \( < z > \) can be approximated by \( n p_h \) i.e., \( < z > \approx n p \). To derive the formulae for \( < z_h > \), we note that the probability that any pair of persons knows each other in a hyper-network (denoted by \( p_h \)) is equivalent to the union of all the probabilities that they connect to each other through any layer \( l \) for all \( l = 1, \ldots, m \). That is:

\[
\begin{align*}
\bar{p}_h &= 1 - (1 - p)(1 - q)^{m-1} \\
\end{align*}
\]

As in a random network, the average degree of a hyper-network also converges to \( n p_h \) when \( n \) is large. Therefore, we have \( < z_h > = n p_h [1 - (1 - q)^{m-1}] \).

The increase in the average degree from an ordinary Poisson network to a hyper-network due to the additional \( m - 1 \) layers is therefore equal to:

\[
\Delta z = < z_h > - < z > = n(1 - p) [1 - (1 - q)^{m-1}] 
\]

When \( q = 0 \), we have \( < z_h > = < z > \)—the average degree of an ordinary Poisson network; and when \( q = 1 \), then \( < z_h > = n \)—a fully connected network. Thus, Poisson and fully connected networks are special cases of hyper-networks. The first and second derivatives of \( \Delta z \) with respect to \( q \) and \( m \) are, respectively:

\[
\begin{align*}
\frac{\partial \Delta z}{\partial q} &= n(m - 1)(1 - p)(1 - q)^{m-2} > 0 \\
\frac{\partial^2 \Delta z}{\partial q^2} &= -n(m - 1)(m - 2)(1 - p)(1 - q)^{m-3} < 0 \\
\frac{\partial \Delta z}{\partial m} &= -n(1 - p) \ln(1 - q)(1 - q)^{m-1} > 0 \\
\frac{\partial^2 \Delta z}{\partial m^2} &= -n(1 - p)[\ln(1 - q)]^2(1 - q)^{m-1} < 0. 
\end{align*}
\]

Therefore, the increment in the average degree is increasing and concave in both \( q \) and \( m \).

To derive a formula for the average distance, we employ a procedure similar to that given in [10]. The difference is that [10] uses the generation function and here we simply use the average degree parameter. In an \( m \)-layer hyper-network, an arbitrary individual \( i \) has on average \( < z_h > \) number of immediate neighbors (of distance 1). These \( < z_h > \) number of nearest neighbors together have on average \( < z_h > ( < z_h > - 1) \) number of immediate neighbors who are of distance 2 away from \( i \). Similarly, these \( < z_h > ( < z_h > - 1)^2 \) number of immediate neighbors who are of distance 3 away from \( i \). The same argument applies to the \( l \) distant neighbors from \( i \), yielding a total number of \( < z_h > ( < z_h > - 1)^{l-1} \) neighbors of distance \( l \) from \( i \). As analyzed in [21], we will reach everyone in the network of \( n \) individuals when \( l \) equals the average distance, hence giving the following equation:

\[
1 + \sum_{l=1}^{\Delta} < z_h > ( < z_h > - 1)^{l-1} = n.
\]
Solving this equation for $\bar{d}$ gives:

$$\bar{d} = \frac{\ln (n - 2(n - 1)/<z_h>)}{\ln (<z_h>-1)}.$$ 

Substituting the relationship $<z_h> = (n - 1)p_h$ to above equation yields:

$$\bar{d} = \frac{\ln (n - 2/p_h)}{\ln (np_h - 1 - p_h)}.$$ 

When $n \to \infty$, this expression is approximately $\bar{d} \approx \ln n/\ln (np_h)$, which is a well-known result in random graph (e.g., [11]).

From Eq.(3), the average distance can be rewritten as:

$$\bar{d}_{m} = \frac{\ln n}{\ln n + \ln (1 - (1 - p)(1 - q)^{m-1})}.$$ 

(4)

We will illustrate in the next section the diversity of social networks that this equation can model, and show its advantages.

VI. HYPER-NETWORK ANALYSIS MAY IMPROVE RANDOM (POISSON) NETWORKS ANALYSIS

Real-world human networks (see, e.g., [10, 12]) tend to be influenced by various institutions that may exert non-random patterns on them, such as organizations, religions, laws, and culture. Therefore, researchers have proposed many enhanced models in the literature to remedy ordinary random graphs (whose degree distributions follow the Poisson distribution). Representative examples include random graphs with arbitrary degree distributions by [10, 13]. They show that in some cases their expressions give accurate predictions compared with real data. However, in other cases, the expressions yield poor estimations. The authors suggest that the poor performance of the expressions could be caused by the presence of unknown social structures in the network and that random graphs fail to account for them.

In the following, we show that such poor performance may due specifically to the presence of multiple roles of participants in the networks. In other words, we submit that the “unknown social structures” are really the other role-based networks super-imposed on the single networks that they studied – that is, single network results were insufficient for studying hyper-networks. For instance, the well-known studies on the movie actor network and the scientist collaboration network in [12-15] reflected only one particular professional relationship: acting together in the movie data set, or collaborating on joint scientific projects in the other. In reality, the actors could also be directors, producers, or Parent-Teacher Association members as well, who made connections through these roles to other actors with whom they never acted. In a similar way, scientists could also be college roommates or friends in their personal lives and/or belong to the same professional societies without ever co-authoring a paper. Ignoring “the other networks” is equivalent to allowing all these persons only one mechanism of interaction in life when in fact that is not the case, resulting in incorrect estimation of the network properties. Obviously, the recommendation here is to explore other attributes in the data set, or additionally other data sets on the same persons, and analyze them together as a hyper-network (using, e.g., Eq. (4)).

Next, we vary $q$ and $m$ in Eq.(4) to obtain a range of distance values. This sensitivity analysis shows that hyper-networks probably can model real social networks better than ordinary random graphs. We construct a Poisson network in such a way that the average vertex-vertex distance is exactly $6$ — i.e., a six-degree of separation. In a Poisson network of $1M$ individuals, this is accomplished if each individual is acquainted with $10$ other individuals, where the average distance (denoted by $\bar{d}_o$) is exactly $\bar{d}_o = \ln 10^6/\ln 10 = 6$

For the hyper-network, we use Eq.(4) with $m = 10$. This value of $m$ is selected so that Eq.(4) is comparable with $\bar{d}_o$ (see Figure 3). We vary $q$ from $0.5\%$ to $100\%$ of $p$, thereby, covering the cases from $p > q$ (e.g., $q/p < 0.005$) to $q/p \approx 1$. Figure 3 depicts the average vertex-vertex distance of the hyper-network (the dotted line) and that of a Poisson network with a fixed six-degree separation (the solid line). As shown, the distance of the hyper-network changes from $5.88$ to $2.93$. This range of distance not only covers the usual six-degree of separation (most studies show a degree of separation between $5$ to $6$) but also the lower-bound result obtain in [8]. We also note that the distance drops quickly when $q$ is small. This implies that acquaintance probability in virtual communities, despite being small, can have a significant impact on distance among people.

![Distance vs. q](image)

Figure 3. Average Distances of Hyper-Network (m=10) and Ordinary Poisson Random Network vs. $q$.

VII. SIMULATE THE DEGREE DISTRIBUTION OF HYPER-NETWORKS: A COMPLETE ANALYSIS

In this section, we study the degree distribution of hyper-networks. We use simulation to grow (or combine) traditional single networks into hyper-networks, and thereby obtain their degree distributions for analysis. The basic research question
is to investigate how the multiple layers of a hyper-network influence its degree distribution. Two factors are of interest in this study: (1) types of connections within a layer and (2) types of connections across layers. Specifically, within a layer, the edges can be random connections (which result in a random graph layer) or preferential attachments (which result in a scale-free network layer). Similarly, the links across layers can also be random connections or preferential attachments. It is these cross-layer connections that have not been studied previously in the literature, and that we believe hold important significance in shaping the behaviors of many human networks. These two types of connections and two types of extensions yield the following four cases of testing:

1. Random connections within each layer and random connections across layers.
2. Preferential attachments within each layer and random connections across layers.
3. Random connections within each layer and preferential attachments across layers.
4. Preferential attachments within each layer and preferential attachments across layers.

In our experiments, we build hyper-networks from the bottom up, mimicking their paths of growth in actual human networks. First, \( m \) layers of networks were created, each with \( n \) nodes, for each of the four cases, separately; and then the layers were combined. The connections among members in each layer followed either Poisson distribution or power law distribution depending on which case it was. To connect the layers, we first linked Layer 2 to Layer 1, resulting in a combined Layer 1-2. We then connected Layer 3 to the combined Layer 1-2, yielding a new combined Layer 1-2-3. This procedure was repeated until Layer \( m \) was linked to the combined Layer 1-2-\ldots-\( m - 1 \). These layers overlay on top of each other, coalescing into a wholly connected hyper-network.

The general method is this: A new layer is linked to the previous combined layer (say, adding Layer \( k \) to combined Layer-1-2-\ldots-\( k - 1 \)) by connecting each node of this new layer to one of the nodes of the combined layer. There are two scenarios in which nodes can be connected between two layers: (1) one-to-one and (2) many-to-one (see Error! Reference source not found.). The one-to-one scenario is to connect a node of Layer \( k \) to one and only one of the nodes (randomly and uniformly) of Layer 1-2-\ldots-\( k - 1 \). Once a node of Layer 1-2-\ldots-\( k - 1 \) has been connected to a node of Layer \( k \), it will no longer be selected in the next round of connection, thus ensuring the connections are one-to-one. In this scenario, each node on a layer is treated as the realization of a character that person is playing for that role (layer). Having multiple characters of the same role for the same person is not allowed in this scenario, but is allowed in the many-to-one scenario (see next paragraph). Because each node in a layer represents only one role of a person, combining one and only one node from each layer across all layers is equivalent to forming a person with full characteristics. Thus, the resulting total number of persons in the hyper-network is still \( n \). However, this may not be true in the many-to-one scenario.

The many-to-one scenario generalizes the one-to-one scenario to allow for multiple characters or more than \( n \) members in the final hyper-network. Specifically, a node of Layer \( k \) selects a target node of Layer 1-2-\ldots-\( k - 1 \) to connect based on certain criteria (in the present experiments, one criterion is the number of current links the target node has, i.e., preferential attachment). Allowing multiple connections across layers provides flexibilities for simulating different network construction mechanisms, such as preferential attachment or other centrality-based mechanisms. In terms of real-world implications of this many-to-one scenario, one can think of a situation where a member or company has several `identities in a particular layer (network). For example, some companies have several sub-companies serving the same industry to hedge against risk, or supplying different types of products for the same target market. Another generalization that the many-to-one scenario provides is that we can now test connecting several actual networks rather than just virtual role networks. Each layer now represents a different actual network with a different set of nodes (such as people, web sites, or companies). The number of nodes in each layer is still \( n \) so that comparisons among the four cases listed above are consistent. This many-to-one scenario can be considered as a situation where a highly popular web site of a particular type (e.g., Facebook.com in the social networking layer) attracts links from multiple other web sites of other types (e.g., restaurant web sites that post their links on Facebook.com).

For control purposes, the hyper-networks in all four cases were created so that the total numbers of links within a layer and across layers were both constant and equal. Keeping these networks at the same number of links makes the comparisons of their degree distributions more comparable.

In summary, the one-to-one scenario was used in Cases 1 and 2 for connecting layers, and the many-to-one was used in Cases 3 and 4 for connecting layers. In addition, uniform distribution was used in the one-to-one scenario, i.e., random connection, and preferential attachment was used in the many-to-one scenario, i.e., the power law behavior.

We now present the experimental results. Figure 5 gives the histograms of degree distributions for the four cases (rows) with different numbers of layers and nodes (columns). The corresponding log-log plots of the degree distributions are presented in Figure 6.

Intuitively, we expect the higher the level of “preference” is, the more skewed the degree distribution would be. Therefore, the power-law behavior should be more obvious in Case 2 than in Case 1 because the former has preferential attachment within each layer; and more apparent in Case 4 than in Case 3 for the same reason. This intuition is confirmed in Figure 5 and Figure 6. Histograms are more skew in the second row (Case 2) as opposed to the first row (Case 1) and similarly in the fourth row (Case 4) as opposed to the third row (Case 3).

The next result is that random connections lead to normal distribution (approximation to the Poisson distribution) as seen in the first and second rows. Random connections can reduce or even remove the power-law behavior as seen in the second
row where the histograms become more normal as \( m \) increases (that is, more layers are connected via random connections). This is likely the result of the Central Limit Theory. Because most real-world datasets do not exhibit normal distribution, it can be inferred that connections across layers in real-world systems are likely to be selective rather than random.

The comparison between Case 2 and Case 3 is concerned with the effects of preferential attachment: applied within a layer (Case 2) versus across layers (Case 3). Histograms of the third row (Case 3) are more skewed than those in the second row (Case 2), suggesting that connections across layers are more influential to the skewedness of the degree distribution.

The power of preferential attachment is evident in the last row, where nodes make connections based on preferential attachment both within and across layers. Therefore, it is like connecting power-law networks by using power-law. While it is still an open question of whether the resulting network is a power-law of a power-law (i.e., a power-law distribution with a power-law exponent), it is clear that such a “double preferential attachment” mechanism gives rise to a higher power distribution than an ordinary preferential attachment network when both networks have the same number of nodes. This can be seen by comparing the first entry of the last row of Figure 5 and the histogram in Figure 7. Both networks have the same \( n \) \((n = 2500)\) and all the nodes of both networks use preferential attachments when making connections. The difference is that the network in Figure 7 has only one layer and all 2500 nodes are connected via preferential attachment within this single layer, while the network in Figure 5 splits these 2500 nodes into 5 layers, each of which is a power-law sub-network, and connects them using preferential attachment (i.e., connects the “power-law” sub-networks in “power-law” fashion). The “power” of the preferential attachment is cascaded when applied across layers, which is made possible by the recognition and creation of hyper-networks.

Most real-world human or social networks in the literature have degree distributions whose nature lies somewhere in between Case 1 and Case 4, that is, closer to Case 2 or, especially, Case 3. While further thorough investigations are needed, one conjecture is that real-world human or social networks are hyper-networks that combine random and preferential networks (layers), with connections across layers being more likely to be preferential. For example, in a citation hyper-network with different layers representing different fields, an (interdisciplinary) article published in a particular field that makes use of techniques of another field is more likely to cite a more influential (highly cited) paper in that second field than to cite a less influential one (i.e., preferential attachment across layers). Therefore, the conjecture is that preferential attachments across layers can transform random networks into something closer to reality.

<table>
<thead>
<tr>
<th>(m, n)</th>
<th>(5, 500)</th>
<th>(20, 500)</th>
<th>(50, 500)</th>
<th>(10, 1000)</th>
<th>(25, 1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td><img src="image1" alt="Degree Distribution" /></td>
<td><img src="image2" alt="Degree Distribution" /></td>
<td><img src="image3" alt="Degree Distribution" /></td>
<td><img src="image4" alt="Degree Distribution" /></td>
<td><img src="image5" alt="Degree Distribution" /></td>
</tr>
<tr>
<td>Case 2</td>
<td><img src="image6" alt="Degree Distribution" /></td>
<td><img src="image7" alt="Degree Distribution" /></td>
<td><img src="image8" alt="Degree Distribution" /></td>
<td><img src="image9" alt="Degree Distribution" /></td>
<td><img src="image10" alt="Degree Distribution" /></td>
</tr>
<tr>
<td>Case 3</td>
<td><img src="image11" alt="Degree Distribution" /></td>
<td><img src="image12" alt="Degree Distribution" /></td>
<td><img src="image13" alt="Degree Distribution" /></td>
<td><img src="image14" alt="Degree Distribution" /></td>
<td><img src="image15" alt="Degree Distribution" /></td>
</tr>
<tr>
<td>Case 4</td>
<td><img src="image16" alt="Degree Distribution" /></td>
<td><img src="image17" alt="Degree Distribution" /></td>
<td><img src="image18" alt="Degree Distribution" /></td>
<td><img src="image19" alt="Degree Distribution" /></td>
<td><img src="image20" alt="Degree Distribution" /></td>
</tr>
</tbody>
</table>

Figure 4. One-to-One Connections (Left) v.s. Many-to-One Connections (Right) Across Layers.

Figure 5. Degree Distributions of Four Cases.
(m, n) = (5, 500)  (20, 500)  (50, 500)  (10, 1000)  (25, 1000)

<table>
<thead>
<tr>
<th>Case</th>
<th>Degree Distribution (log-log)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td><img src="image1" alt="Degree Distribution" /></td>
</tr>
<tr>
<td>Case 2</td>
<td><img src="image2" alt="Degree Distribution" /></td>
</tr>
<tr>
<td>Case 3</td>
<td><img src="image3" alt="Degree Distribution" /></td>
</tr>
<tr>
<td>Case 4</td>
<td><img src="image4" alt="Degree Distribution" /></td>
</tr>
</tbody>
</table>

Figure 6. Log-Log Plots of Degree Distributions of Four Cases; Straight Lines Indicate Power Law.

Figure 7. Degree Distribution of a 1-Layer Preferential Attachment Network with \( \eta = 2,500 \).

VIII. SUMMARY AND CONCLUSIONS

This paper recognizes modern service as an effort characterized by hyper-networking of customers, providers, and resources for value cocreation. It develops a new analysis of this hyper-networking to investigate how to improve the quality and productivity of service value networks, and corroborates the analysis with empirical evidence shown in the field of e-commerce. The hyper-network model is established to substantiate the analysis. The hyper-network model adds new basic results to the field of network science by extending the previous two-dimensional models to span any dimensions. A simulation study suggests that the new results may also hold the promise of improving the traditional analysis of social networking, such as explaining some well-known underestimation of acquaintances in the literature.

In particular, Sections I – III reveal the hyper-networking nature of e-commerce, social networking, and physical products that incorporate services. The analysis uniquely explains how the dynamics of scaling service value networks may continue to drive the field. Sections IV – VII, then, establish the hyper-network model. They show that randomly adding connections among members in a single network amounts to building a hyper-network. Since this practice is common with Internet enterprises, the analysis suggests that hyper-networks are ubiquitous in actuality. They also provide expressions for estimating the average degree and average distance of the hyper-networks. These expressions are relatively simple for application, and yet effective at analyzing many human networks, including social networks.

The simulation conducted confirms a general propensity of hyper-networks to reduce the average distance between nodes, as compared to previous results with two-dimensional graphs. The study also shows that the reduction applies to both random and scale free models and hence could be invariant to the underlying structure of the network. Finally, the simulation reveals possible patterns of degree distribution in various hyper-networks, as the result of combining random and preferentially connected single networks.

The hyper-network model may be further expanded along a number of directions. A promising opportunity is to include behavioral complexities such as feelings and intensity in the definition of connections, as suggested in Section III. With this extension, the model should be ready to help analyze such complex issues as trust [28] and multidimensional social network [29]. New research can also focus on empirical studies. These new studies may uncover hyper-networks in large-scale empirical data sets used for social networks analysis [30]. The distance formulae promise to help analyze how the intertwined nature of roles may have obscured the previous two-dimensional analyses. Future studies may also discover hyper-hubs (or “value wormholes” [5]) that connect layers of networks and facilitate centrality of nodes. They may also expose more hyper-network connection laws and thereby...
help develop better connection paths or strategies to promote hyper-networking by using these laws, e.g., suggesting directions for e-marketing and new business design.

Looking forward, one may observe that a global hyper-network (of hyper-networks) is indeed taking shape. The forces of globalization assisted by worldwide cyber-infrastructure may steadily overtake the traditional cultural divides and achieve transcendental connections at a global scale. Although individual disciplines and applications will focus only on the particular aspects and/or subsets of the global hyper-network that concern them, the very awareness of such hyper-networks may prompt researchers to broaden their visions of study. After all, engineering, economics, and all other human activities reflect hyper-networking of humans and resources. In this light, we might further understand the mechanisms that reconcile the solitary nature of human beings with their socialization that develops humanity.

REFERENCES


